MPZ 3231 - Assignment 3

(1) Using the first and second derivatives, find the local extreama of the following function and disscuss the concaving. Determine the points of inflection and find the vertical and horizontal asymptotes for the following functions. Sketch the graphs of y.

(a) $y = \frac{x}{x^2-1}$ (b) $y = \frac{2x+4}{x-1}$ (c) $y = \frac{x^2-4}{x+1}$

(2) Evalate each of the following limits using L'hospitals rule

(a) $lim_{x\to\infty^+} \frac{x}{(\ln x)^3+2x}$ (b) $lim_{x\to o} \left(\frac{1}{x} - \frac{1}{\sin x}\right)$ (c) $lim_{x\to 2} \left(\frac{x}{2}\right)^{\frac{1}{x-2}}$

(d) $lim_{x\to\frac{\pi}{4}}(\tan x - 1) \sec 2x$ (e) $lim_{x\to o} \left(\frac{1+\tan x}{1+\sin x}\right)^{\frac{1}{\sin x}}$ (f) $lim_{x\to o} \left(\frac{\ln(1+x)-\sin x}{x \sin x}\right)$

(3) (a) if $v = u^2v$ and $u = e^{x^2-y^2}$; $v = \sin(x y^2)$ Find $\frac{\partial v}{\partial x}$, $\frac{\partial v}{\partial y}$

(b) Prove that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$

If z is a function of x and y and $x = e^u + e^{-v}$; $y = e^{-u} + e^v$

(c) Prove that $y\frac{\partial v}{\partial y} - x\frac{\partial v}{\partial x} = x^2v^3$; If $v = (1 - 2xy + y^2)^{(-1/2)}$

(d) Show that $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2$ when u is a function of x , y where

$x = r\cos\theta, y = r\sin\theta$

(4) (i) Find the perpendicular distance between the planes $3x + 6y + 2z = 22$ and $3x + 6y + 2z = 27$. Draw a figure.

(ii) Find the equation of the plane passing through the line of intersection of the planes $2x - y = 0,\ 3z - y = 0$ and perpendicular to the plane $4x + 5y - 3z = 8$.

(iii) Find the value of α and β so that the planes: $x + 2y + z = \beta, 3x - 5y + 3z = 1$ and $2x + 7y + \alpha z = 8$ intersect in

(a) one point (b) a line (c) three distinct and parallel lines

(5) (i) (a) Find the equations of the perpendicular drawn from the point (5,9,3) to the line

$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$. Find also the co-ordinates of the foot of the perpendicular.

(b)Find k so that the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-5}{1} = \frac{z-6}{-5}$ may be perpendicular to each other.

(ii) (a) Write the equation of the line through the point $(2,1,-2)$ and perpendicular to the plane $3x - 5y + 2z + 4 = 0$.

(b) Find the length and equations of the line of shortest distance between the lines $\frac{x+3}{-4} = \frac{y+6}{3} = \frac{z}{2}$ and $\frac{x+2}{-4} = \frac{y}{1} = \frac{z-7}{1}$.

(iii) Find the equations of the tangents to the curve $4x^2 - 2y^2 - 3xy + 2x - 3y - 10 = 0$ which are parallel to the line $x - y + 5 = 0$.

(iv) Find the equations of the tangents to the hyperbola $xy = 2$ which are perpendicular to the line $x - 2y = 7$.